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VARIATION OF GAS VELOCITY IN A NORMAL IONIZING SHOCK WAVE AND THE PROBLEM OF THE CONDUCTIVE PISTON

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The variation of gas velocity in an ionizing shock wave propagating along an initial magnetic field (a normal ionizing wave), when the gas magnetic viscosity is considerably greater than the remaining dissipative coefficients, is investigated in this paper. Obtained results are used for the derivation of solution of the problem of motion of a conductive piston. Similar investigations in which the wave front orientation with respect to the magnetic field was arbitrary, were the subject of paper [1].

An analysis of the limit case of normal ionizing shock waves is of interest in view of the numerous experimental investigations of such waves [2 and 3], and also due to the presence of a number of singularities in its solution as compared with the general case. Variations of the magnetic field profile, of density and other parameters in a supersonic normal ionizing shock wave and in the subsequent MHD rarefaction wave were computed in paper [4] in connection with the problem of discharge. It was assumed there that in a varying magnetic field the ionizing wave becomes an ionizing Jouguet wave. It will be shown in the following that this assumption is correct.

In the case of normal ionizing waves here considered the flow is a plane one, i.e. the gas velocity and the magnetic field lie in one and the same plane drawn through the normal to the wave front. We introduce the system of coordinates x, y, z with the x-axis directed along a normal to the wave, and the magnetic field component behind the wave $H_{z2} = 0$. Let in this coordinate system u and v be the velocity variations along the x- and y-axes.

In the case under consideration intermediate ionizing shock waves are absent, i.e. out of the five wave types [5] three only are possible, viz. supersonic fast, supersonic and subsonic slow ionizing waves. In supersonic ionizing waves the magnetic field, and consequently also the gas velocity tangent component do not vary. In the uv-plane points lying along the u-axis to the right of u' correspond to a fast wave.

Let $a_0' = a_0' \sqrt{4\pi\rho_1} / H_x$ be the dimensionless velocity of sound in front of the wave. Then $u' = u^*$ when $a_0 > 1$ where u^* is the velocity variation of the wave behind which the temperature becomes critical, i.e. it belongs to the common boundary of areas with zero and nonzero electrical conductivities. For $a_0 < 1$

$$u' = \max\,[u^*, u_E] \qquad \left(u_E = \frac{2H_x(1 - a_0^2)}{\sqrt{(\gamma - 1)4\pi\rho_1}\ \sqrt{\gamma + 1 - 2a_0^2}}\right)$$

Here the expression of u_E relates to a perfect gas. Fig. 1 and 2 show respectively the case of $a_0 < 1$ and $a_0 > 1$.

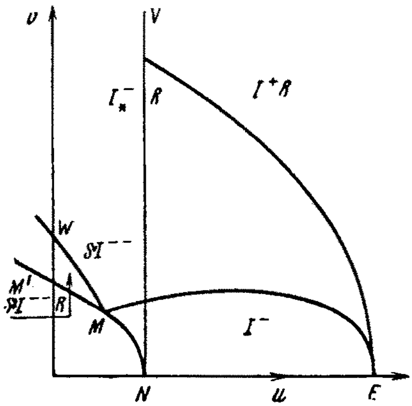

Fig. 1

Supersonic slow ionizing waves occur for $a_0 < 1$ and $u^* < u_E$ only. As the variation of the two velocity components in these waves is within certain limits arbitrary, a two-dimensional area corresponds to them in the uv-plane. Because in the case here considered this area is symmetric with respect to the u-axis, only that of its part which corresponds to $v > 0$ has been shown on Fig. 1. Its boundary consists of segments of lines along which one of the following conditions is fulfilled:

1) the Jouguet condition, i.e. behind the wave the gas velocity relative to the ionizing wave is equal to the slow magnetosonic velocity $u_2 = a_-$ (curve ME),

2) the ionizing wave velocity is equal to the velocity of the gas-dynamical wave behind which the atate (of the gas) becomes critical (curve NM).

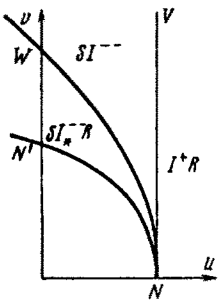

Fig. 2

A subsonic slow ionizing wave occurs for $u^* < u_E$, and is always preceded by a gas-dynamical shock wave behind which the state of the gas becomes critical. For the convenience of subsequent exposition the velocity variation of the gas traversed by a system of two waves, viz. the gas-dynamical shock wave and the subsonic slow ionizing wave is shown on Fig. 1 and 2. For $a_0 < 1$ this velocity variation belongs to area $WMNV$ (Fig. 1), and to area WNV for $a_0 > 1$ (Fig. 2), and also to areas symmetric with respect to the u-axis. The boundary of this area consists of lines along which one of the following conditions is fulfilled:

1) the ionizing wave velocity is zero (NV),

2) velocity $u_2 = a_-$ (the Jouguet line), MW on Fig. 1, and NW on Fig. 2,

3) the ionizing wave propagation velocity relative to the gas-dynamical shock wave is zero (for $a_0 < 1$ only, MN on Fig. 1).

We shall now consider the self-similar problem of the gas flow generated by the motion of a conductive piston. Let at the inital instant $t = 0$ a perfectly conductive flat piston begin to move at constant velocity $U = ue_x + ve_{\|}$ from position $x = 0$. At the initial instant there is in area $x > 0$ nonconductive gas of constant density and pressure at rest in a homogeneous electrical $E = Ee_z$ and magnetic $H = H_x e_x$ fields which initially are perpendicular to the piston face. Let us assume that the piston motion generates an ionizing shock wave behind which the gas electrical conductivity is infinitely great.

The gas motion is to be determined. The boundary condition at the piston (face) is the coincidence of the gas and piston velocities if the gas contacts the piston, or when there is a vacuum between the piston and gas by the linear relationship $(\mathbf{U}-\mathbf{U}_r) \times \mathbf{H} = 0$ [6].

We note that the problem of electrical discharge is a particular case of the piston problem as formulated above, when $u = 0,\ v = cE / H_x$.

The stated problem is self-similar, and its solution consists of surfaces of discontinuity and simple waves separated by areas of constant values of all parameters. An electromagnetic wave propagates in front.

If the initial electric field is small in comparison with the magnetic one, and $U/c \ll 1$, then the magnetic field changes in the electromagnetic wave may be neglected, i.e. we can consider the magnetic field in front of the ionizing wave as being equal to the initial one. Hence, the electric field in front of an ionizing wave is determined by the solution of the problem, while the electromagnetic wave need not to be considered.

We shall represent piston velocities in the vu-plane by dots. This plane then becomes subdivided into a number of areas to every point of which correspond solutions consisting of the same combination of waves. The form of these areas is qualitatively shown on Fig. 1 and 2 for the case of $a_0 < 1$ and $a_0 > 1$ resepectively.

Each area has been marked by the wave combination representing solutions for piston velocities appertaining to that area. The following notations have been used: I^+. I^- for supersonic fast and slow ionizing waves, I^{--} for subsonic slow ionizing wave, $I_*^- I_*^{--}$ for slow ionizing Jouguet waves, R for magnetohydrodynamic slow rarefaction wave, and S for gas-dynamical shock wave behind which the state of gas is critical.

We note that to the left of line $M'MN$ on Fig. 1, and of line $N'N$ on Fig. 2 there are no solutions with ionizing waves. Two solutions with supersonic and subsonic ionizing waves are possible in area $M'MNV$ of Fig. 1, and in area $N'NV$ of Fig. 2. We shall point out the basic singularities of solutions containing a normal ionizing wave as compared to the general case.

1. Solutions with intermediate ionizing waves are absent.

2. There are no solutions containing Alfven and magnetohydrodynamic slow shock waves. In an area in which a solution with subsonic ionizing waves is possible a solution with supersonic waves is also possible. In that case, however, solutions with subsonic waves correspond to the second leaf of solutions [1], and are evidently unstable, so that a solution containing supersonic waves is obtained.

3. The change of sign of v in the solution results in the change of signs of the gas transverse velocity component and of that of the magnetic field only.

We note that solutions with supersonic slow ionizing waves are absent when $a_0 > 1$ or $u^* > u_E$. It follows from Fig. 1 and 2 that the solution of the discharge problem with supersonic waves consists of an ionizing wave and rarefaction wave, with the ionizing wave being either a slow ionizing Jouguet wave, or a fast ionizing one, depending on initial values and the applied electric field.

This proves the assumption made by Taussing. We note that depending on the conditions of experiments either of the two modes indicated here were observed in experiments with electrical discharge in a tubular ring in the presence of a magnetic field.

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ON THE EVOLUTIONARITY OF EQUATIONS OF MAGNETOHYDRODYNAMICS TAKING THE HALL EFFECT INTO ACCOUNT

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It has been established that the system of equations of magnetohydrodynamics for a non-dissipative plasma plane flow across a magnetic field will not be evolutionary if the Hall effect is taken into account.

The numerical solution of the problem of plasma flow in a coaxial channel with the Hall effect taken into account had disclosed [1] the flow instability which is the more pronounced the stronger the Hall effect is. This instability of a nonstationary plasma flow develops in the vicinity of the anode, and has the character of an explosion in which a sharp rise of the current density and particle velocity takes place. An experimental investigation of such flows [2] had disclosed the appearance of considerable jumps of potential in the anode vicinity with the flow itself becoming unstable, resulting in the so-called "current attachment" and severe anode erosion.

These results and observations make it desirable to carry out a mathematical analysis of the two-dimensional plasma flow stability taking into account the Hall effect. The present paper is devoted to a comparatively simpler, but important result which has to be kept in mind in detailed investigations on this subject, namely that the equations of non-dissipative magnetohydrodynamics for a two-dimensional plane flow across a magnetic field are nonevolutionary when the Hall effect is taken into account. The term nonevolutionary (or incorrectly) [3] is understood here to mean the instability of solution of the Cauchy problem with respect to high frequency perturbations which increase arbitrarily fast. This result is also true for axisymmetric flows acted upon by an azimuthal magnetic field, in so far as these may be considered to be locally plane.

The flow of an inviscid and non-heat-conductive plasma in the presence of the Hall effect is defined (in a dimensionless form and with the usual notations [1]) by Eqs.